

Renormalized Wick expansion for a modified PQCD

Alejandro Cabo Montes de Oca

*International Center for Theoretical Physics, Strada Costiera 111, Trieste, Italy and
Group of Theoretical Physics, Instituto de Cibernética,
Matemática y Física, Calle E, No. 309, Vedado, La Habana, Cuba*

The renormalization scheme for the Wick expansion of a modified version of the perturbative QCD introduced in previous works is discussed. Massless QCD is considered, by implementing the usual multiplicative scaling of the gluon and quark wave functions and vertices. However, also massive quark and gluon counter-terms are allowed in this mass less theory since the condensates are expected to generate masses. A natural set of expansion parameters of the physical quantities is introduced: the coupling itself and to masses m_q and m_g associated to quarks and gluons respectively. This procedure allows to implement a dimensional transmutation effect through these new mass scales. A general expression for the new generating functional in terms of the mass parameters m_q and m_g is obtained in terms of integrals over arbitrary but constant gluon or quark fields in each case. Further, the one loop potential, is evaluated in more detail in the case when only the quark condensate is retained. This lowest order result again indicates the dynamical generation of quark condensates in the vacuum.

PACS numbers: 12.38.Aw;12.38.Bx;12.38.Cy;14.65.Ha

I. INTRODUCTION

The relevance of properly understanding QCD is difficult to overestimate. However, due to the known difficulties associated to its strong interaction properties, the predictions of QCD are also extremely far from a satisfactory knowledge. Thus, the investigation of the properties of the theory should be attacked from all possible angles, as it has been undertaken along many years [1, 2, 3, 4, 5, 6, 7, 8, 9]. The motivations for considering this work (and a few of previous ones done in the theme in conjunction with other colleagues [10, 11, 12, 13, 14, 15, 16, 17, 18]) can be resumed as follows: Firstly, to consider that within modern views in high energy physics, the masses are normally searched to appear as generated by a spontaneous or dynamical symmetry breaking in starting mass less theories. Then, the circumstance that the QCD conveys the strongest forces of Nature, in combination with the fact that the mass less version of the theory has not a definite mass parameter, directly leads to the physical relevance of examining the scales of dynamical mass allowed by symmetry breaking processes in mass less QCD [6, 19]. In former works we have got indications about the possibility of generating masses in QCD [12, 18]. In [12] modified Feynman rules were employed for evaluating the quark masses from the Dyson equation in which the simplest corrections to the self-energy determined by the condensates were retained. For this purpose the gluon condensate parameter C_g was evaluated by fixing the mean value of the gluon Lagrangian (a quantity which in the proposed picture is non-vanishing in the lowest approximation) to its estimated value in the literature. The result for initially massless quarks, surprisingly gave a value of one third of the proton mass [12]. That is, a prediction of the constituent quark masses followed.

Motivated by this result, in Ref. [18] we considered similar evaluations assuming also the presence of a quark condensates for any flavour C_f , $f = 1, 2 \dots 6$ and for the gluons C_g . In this case, by properly selecting the coefficients C_f and C_g , it was possible to obtain the quark masses as singularities of the propagator for the six quarks, also in the simplest approximation. Thus, the question emerged about the possibility for those condensate values to be generated as the result of a dynamical symmetry breaking in mass less QCD. In Ref. [14] this issue was started to be considered by evaluating particular summations of one loop diagrams. The results were positive in the direction of supporting the generation of the gluon as well as the quark condensates. However, the instability of the potential at zero value signaling the production of the quark condensate, had not a bounded from below form. This fact did not allowed to predict a concrete mean value of the condensate to be approached by the system after stabilization. However, this result could be a consequence of first evaluations in a scheme in which the renormalization was not yet implemented or of the low order corrections which were calculated.

Therefore, in this work we start considering the renormalization of the proposed expansion and its application to evaluate the first order contributions to the Effective Action in terms of the quark and gluon condensates. For this purpose, we are already supported by the discussion in Refs. [13] in which the gauge invariance of the proposed Feynman expansion was argued. Also and importantly, a procedure was devised for eliminating the apparent singularities appearing in the graph expansion due to the presence of Dirac Delta functions of the momenta [13, 20].

Here we start by introducing the renormalization prescription in the Euclidean version of the generating function of the Green functions $Z = \exp(W)$ depending of the external sources, which also will be a function of the condensate parameters for quarks and gluons. Multiplicative renormalization is implemented for the fields and coupling constants in the Wick expansion. However, it should be noticed that mass counter-terms will be also added, although the bare theory before the adiabatic connection of the interaction is being assumed mass less QCD. This assumption is essential: we are considering that the mass counter-terms will be automatically generated by the interactions, although they can not be implemented by the multiplicative scaling of the fields and parameters. The Effective Action is then introduced as usual, as the Legendre transform over the external sources in favour of the mean values of the quantum fields. The effective potential determined by it, is also a function of the gluon and quark condensates and naturally it should show a minimum with respect to these quantities at the ground state. Here, the fields values are fixed to vanish in the ground state assuming the Lorentz invariance of the vacuum from the start. All the generating functionals defined are considered as expanded in power series of the coupling constant g , and the two defined mass parameters $m_q = (g^2 C_q)^{\frac{1}{2}}$ and $m_g = (g^2 C_g)^{\frac{1}{2}}$. This reordering of the expansion of the physical quantities allows to implement the dimensional transmutation effect in the considered mass less QCD. The generating functional Z is obtained in a form that resumes the effect of the condensates as generated by Gaussian weighted averages over constant homogeneous background gauge fields for gluons and quarks, respectively. These formulae are expected to be considered for detailed calculations elsewhere. Finally, the lowest order correction to the potential is evaluated in more detail for the case of the single presence of the quark condensate. The dependence of the potential indicates a tendency to the dynamical generation of the condensate in the lowest order.

The work will proceed as follows. In Section 2, the Feynman expansion for QCD in Euclidean variables and the conventions to be used, will be described. Next, in the Section the renormalized generating functional expressing the Feynman expansion in momentum space is written. Section 3 considers the calculation of the Effective Potential in zero order in g for the case of the single presence of the quark condensate. Finally, the results are reviewed and commented in the Summary.

II. GENERATING FUNCTIONAL $Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}]$

As remarked in the Introduction the purpose of the this Section is to present a renormalized version of the proposed perturbative expansion. For this aim the starting action for mass less QCD in Euclidean variables will be taken in the form

$$S = \int dx \left(-\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - \frac{1}{2\alpha} \partial_\mu A_\mu^a \partial_\nu A_\nu^a - \bar{\Psi}_q^i i \gamma_\mu D_\mu^{ij} \Psi_q^j - \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right), \quad (1)$$

where the field intensity, and covariant derivatives follow the conventions

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c, \\ D_\mu^{ij} &= \partial_\mu \delta^{ij} + ig A_\mu^a T_a^{ij}, \quad D_\mu^{ab} = \partial_\mu \delta^{ab} + g f^{abc} A_\mu^c, \\ \{\gamma_\mu, \gamma_\nu\} &= -2\delta_{\mu\nu}, \quad [T_a T_b] = i f^{abc} T_c. \end{aligned} \quad (2)$$

The Fourier decomposition for any field, i.e. the gauge one, will assumed in the form

$$\begin{aligned} A_\mu^b(x) &= \int \frac{dk}{(2\pi)^D} \exp(ik_\mu x_\mu) A_\mu^b(k), \\ A_\mu^b(k) &= \int dx \exp(-ik_\mu x_\mu) A_\mu^b(x). \end{aligned}$$

Then, the renormalized Green's functions generating functional including the gluon and quark condensate param-

eters C_q and C_g as discussed in Ref. [13], is expressed as follows

$$\begin{aligned}
Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \frac{I[j, \eta, \bar{\eta}, \xi, \bar{\xi}]}{I[0, 0, 0, 0]}, \\
I[j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \exp(V^{int}[\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{\eta}}, \frac{\delta}{\delta \eta}, \frac{\delta}{\delta \bar{\xi}}, \frac{\delta}{\delta \xi}]) \times \\
&\exp(\int \frac{dk}{(2\pi)^D} j(-k) \frac{1}{2} D(k) j(k)) \times \\
&\exp(\int \frac{dk}{(2\pi)^D} \bar{\eta}(-k) G_q(k) \eta(k)) \times \\
&\exp(\int \frac{dk}{(2\pi)^D} \bar{\xi}(-k) G_{gh}(k) \xi(k)),
\end{aligned} \tag{3}$$

in which the only changes with respect to the functional associated to the usual perturbative QCD appear in only two of the three free propagators of the expansion, the quark and the gluon ones[10, 11, 12, 13, 14, 15, 16, 17, 18]:

$$\begin{aligned}
D_{\mu\nu}^{ab}(k) &= \delta^{ab}(\frac{1}{k^2}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\theta_N(k) + C_g^b \delta^D(k) \delta_{\mu\nu}), \\
G_f^{ij}(k) &= \delta^{ij}(\frac{\theta_N(k)}{m_f + \gamma_\mu k_\mu} + C_q^b \delta^D(k) I), \\
G_{gh}^{ab}(k) &= \delta^{ab} \frac{\theta_N(k)}{k^2}.
\end{aligned} \tag{4}$$

That is, the quark and gluon propagators now include the condensation effects through the C_q and C_g parameters respectively. However, an important additional change is also present in (4). The usual Feynman propagators are regularized in the neighborhood of zero momentum by multiplying them by

$$\theta_N(k) = \theta(\sigma - |k|), \quad |k| = (k_\mu k_\mu)^{\frac{1}{2}}, \tag{5}$$

where θ is the Heaviside function and σ is an infinitesimal momenta cutoff, that we will call as the Nakanishi parameter. As discussed in [13], this regularization naturally arise when gauge theory propagators are properly defined. Here we assumed that upon passing to the Euclidean version of the theory, the prescription can be translated to eliminate a neighborhood of the momentum space near the zero momentum point. This infrared regularization in combination with a dimensional regularization rule for products of Dirac Delta functions evaluated at zero momenta, allowed in Ref. [13] to eliminate the singularities appearing in the perturbative series by the presence of products of Delta functions evaluated at zero momenta and of them with propagators evaluated also at vanishing momenta.

The vertices defining the interaction in the Wick expansion formula (3) have the decomposition

$$\begin{aligned}
V^{int} &= V_g^{(1)} + V_g^{(2)} + V_q^{(1)} + V_{gh}^{(1)} + \\
&(Z_1 - 1)V_g^{(1)} + (Z_4 - 1)V_g^{(2)} + (Z_{1F} - 1)V_q^{(1)} + \\
&(\tilde{Z}_1 - 1)V_{gh}^{(1)} + \\
&V_g^{(0)} + V_q^{(0)} + V_{gh}^{(0)},
\end{aligned} \tag{6}$$

where the superindices indicate the order the coupling constant associated to the original vertices in the bare action. As usual the unrenormalized bare fields (signaled with a superindex b) will be related to their renormalized counterparts through the factors "Z" as

$$\begin{aligned}
A_\mu^b &= Z_3^{\frac{1}{2}} A_\mu, \\
\Psi_q^b &= Z_2^{\frac{1}{2}} \Psi_q, \quad \bar{\Psi}_q^b = Z_2^{\frac{1}{2}} \bar{\Psi}_q, \\
\chi^b &= Z_3^{\frac{1}{2}} \chi, \quad \bar{\chi}^b = Z_3^{\frac{1}{2}} \bar{\chi},
\end{aligned} \tag{7}$$

with the also usual correspondence between the sources

$$\begin{aligned}
j_\mu^b &= Z_3^{-\frac{1}{2}} j_\mu, \\
\eta_q^b &= Z_2^{-\frac{1}{2}} \eta_q, \quad \bar{\eta}_q^b = Z_2^{-\frac{1}{2}} \bar{\eta}_q, \\
\xi^b &= Z_3^{-\frac{1}{2}} \xi, \quad \bar{\xi}^b = Z_3^{-\frac{1}{2}} \bar{\xi}.
\end{aligned} \tag{8}$$

It should be underlined that the bare condensate parameters C_q^b and C_g^b appear in the free propagators because, these constants had not been expanded yet in their renormalized and counterterm contributions. This will be performed later within what we think is a more convenient representation for this purpose. It is also assumed that the scale parameter μ of dimensional regularization links the dimensional coupling g with its dimensionless value g_o as

$$g = g_o \mu^{2-\frac{D}{2}} = g_o \mu^\epsilon. \quad (9)$$

The expressions for each of the vertices entering V^{int} in (6) are given in the Appendix A. Let us examine more closely the vertex $V_q^{(0)}$ associated to the mass and wave function renormalization of the bare theory. It should be first recalled that the modified expansion under consideration was motivated by deriving the Wick expansion for a mass less QCD in which the mass parameter is absent. Then, the renormalization procedure being investigated is consequently assumed to represent the physics of an adiabatic connection of the interaction from an originally mass less theory. Therefore, we estimate as the most natural procedure to fix the bare masses of gluons and quarks as vanishing. However, it is clear that since the theory has been argued to generate mass [12, 18], the connection of the interaction should be expected to produce mass counterterms in the renormalized action. These terms will also assumed to appear among the quark counterterm vertices $V_q^{(0)}$ in Appendix A with the form:

$$V_q^{(0)}\left[\frac{\delta}{\delta\bar{\eta}}, \frac{\delta}{\delta\eta}\right] = \int \int dk_1 dk_2 (2\pi)^D \delta^D(k_1 + k_2) \times \frac{\delta}{\delta\eta(k_2)} ((Z_2 - 1)k_{1\mu}\gamma_\mu + \delta m(g_o, m_q, m_g)) \frac{\delta}{\delta\bar{\eta}(k_1)},$$

although a non vanishing δm mean a break of pure multiplicative renormalization. This seems to be not a complication since multiplicative; renormalization is known to be broken when the theory has no symmetries that enforces the vanishing of allowed counterterms having no counterpart in the original bare action.

Now, the source terms in the gluon condensate propagator can be represented as a Gaussian integral over constant and homogeneous gauge boson fields as follows

$$\begin{aligned} \exp\left[\left(\int \frac{dk}{(2\pi)^D} j_\mu^a(-k) C_g^b \delta(k) j_\mu^a(k)\right)\right] &= \exp\left(\frac{C_g^b}{(2\pi)^D} j_\mu^a(0) j_\mu^a(0)\right), \\ &= \frac{1}{(2\pi)^{(N^2-1)D} \mathcal{N}_g(Z_3)} \int d\alpha_\mu^a \exp\left[\left(\frac{Z_3-1}{Z_3}\right) \frac{\alpha_\mu^a \alpha_\mu^a}{2}\right] \times \\ &\quad \exp\left[-\frac{\alpha_\mu^a \alpha_\mu^a}{2} + \left(\frac{2C_g}{(2\pi)^D}\right)^{\frac{1}{2}} j_\mu^a(0) \alpha_\mu^a\right], \\ &= \frac{1}{(2\pi)^{(N^2-1)D} \mathcal{N}_g(Z_3)} \exp\left[\frac{(Z_3-1)}{2} \left(\frac{2C_g}{(2\pi)^D}\right)^{-1} \frac{\partial^2}{2\partial j_\mu^a \partial j_\mu^a}\right] \times \\ &\quad \int d\alpha_\mu^a \exp\left(-\frac{\alpha_\mu^a \alpha_\mu^a}{2} + \left(\frac{2C_g}{(2\pi)^D}\right)^{\frac{1}{2}} j_\mu^a(0) \alpha_\mu^a\right), \end{aligned} \quad (10)$$

where \mathcal{N}_g is a normalization constant which cancels with a similar one appearing in the normalizing factor $I[0, 0, 0, 0]$ in (3). Here, the process of, let say, "translating" the renormalization part $(Z_3 - 1)C_g$ of the gluon condensate to the counterterms is started by expressing the terms containing those parts as an exponential of a quadratic form in the derivatives over the sources.

For quarks, an analog formula in terms of interaction over anti-commuting fermion fields will be employed. It has the form

$$\begin{aligned} \exp\left(\int \frac{dk}{(2\pi)^D} \bar{\eta}_u^i(-k) C_q^b \delta(k) \eta_u^i(k)\right) &= \exp\left(\frac{C_q^b}{(2\pi)^D} \bar{\eta}_u^i(0) \eta_u^i(0)\right) \\ &= \frac{1}{\mathcal{N}_q(Z_2)} \int d\bar{\chi}_u^i d\chi_u^i \exp\left[\frac{(Z_2-1)}{Z_2} \bar{\chi}_u^i \chi_u^i\right] \\ &\quad \exp\left[-\bar{\chi}_u^i \chi_u^i + \left(\frac{C_q}{(2\pi)^D}\right)^{\frac{1}{2}} (\bar{\eta}_u^i(0) \chi_u^i + \bar{\chi}_u^i \eta_u^i(0))\right], \\ &= \frac{1}{\mathcal{N}_q(Z_2)} \exp\left[\frac{(Z_2-1)}{Z_2} \left(\frac{C_q}{(2\pi)^D}\right)^{-1} \frac{\partial^2}{-\partial \eta_u^i(0) \partial \bar{\eta}_u^i(0)}\right] \times \\ &\quad \int d\bar{\chi}_u^i d\chi_u^i \exp\left(-\frac{\bar{\chi}_u^i \chi_u^i}{2} + \left(\frac{C_q}{(2\pi)^D}\right)^{\frac{1}{2}} (\bar{\eta}_u^i(0) \chi_u^i + \bar{\chi}_u^i \eta_u^i(0))\right), \end{aligned} \quad (11)$$

in which again the appearing factor \mathcal{N}_q will be cancelled by a similar one appearing in $I[0,0,0,0]$. In the above expressions the condensate parameters have been naturally decomposed in the way

$$\begin{aligned} C_g^b &= Z_3 C_g = (Z_3 - 1)C_g + C_g, \\ C_q &= Z_2 C_q = (Z_2 - 1)C_q + C_g. \end{aligned}$$

It can be recalled that the condensates were created by the action over the vacuum of exponential of quadratic forms for the bare gluon and quark creation operators [11, 13]. Therefore, the renormalization of these operators implies that the bare parameters should be related to the renormalized ones through the same Z_3 or Z_2 constants for gluons and quarks respectively. It can be noted that within the quadratic form defining the gluon condensate quadratic terms in the ghost fields also appeared. However, since the ghosts have the same renormalization constant that the gluons, the Z_3 proportionality between the bare and renormalized gluon condensates should remain valid.

The following relationships help to transform the functional derivatives over the sources, when integrated around zero momentum within the Nakanishi neighborhood, as usual derivatives over the zero momentum components of these sources.

$$\begin{aligned} \int dk \frac{\delta}{\delta j(k)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}] \theta_N(k) &= \frac{\partial}{\partial j(0)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}], \\ \int dk \frac{\delta}{\delta \eta(k)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}] \theta_N(k) &= \frac{\partial}{\partial \eta(0)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}], \\ \int dk \frac{\delta}{\delta \bar{\eta}(k)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}] \theta_N(k) &= \frac{\partial}{\partial \bar{\eta}(0)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}], \\ \int dk \frac{\delta}{\delta \xi(k)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}] \theta_N(k) &= 0, \\ \int dk \frac{\delta}{\delta \bar{\xi}(k)} F[j, \eta, \bar{\eta}, \xi, \bar{\xi}] \theta_N(k) &= 0. \end{aligned}$$

After employing the above relations, the Z functional can be represented in a form where the effects of the condensates are conveyed by the newly incorporated gluon and quark constant and homogeneous fields $\alpha, \bar{\chi}$ and χ (below, they will be named as the *auxiliary fields*). The expression for Z is

$$\begin{aligned} Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \frac{I[j, \eta, \bar{\eta}, \xi, \bar{\xi}]}{I[0, 0, 0, 0]}, \\ I[j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \frac{1}{\mathcal{N}} \int \int d\alpha d\bar{\chi} d\chi \exp[-\bar{\chi}_u \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}] \\ &\exp[\widehat{V}^{int}[\frac{\delta}{\delta j} + (\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}} \alpha, \frac{\delta}{\delta \bar{\eta}} + (\frac{C_q}{(2\pi)^D})^{\frac{1}{2}} \chi, \frac{\delta}{-\delta \eta} + (\frac{C_q}{(2\pi)^D})^{\frac{1}{2}} \bar{\chi}, \frac{\delta}{\delta \bar{\xi}} - \frac{\delta}{\delta \xi}, \alpha, \bar{\chi}, \chi]] \times \\ &\exp[\int \frac{dk}{(2\pi)^D} j(-k) \frac{1}{2} D^F(k) j(k)] \times \\ &\exp[\int \frac{dk}{(2\pi)^D} \bar{\eta}(-k) G_q^F(k) \eta(k)] \times \\ &\exp[\int \frac{dk}{(2\pi)^D} \bar{\xi}(-k) G_{gh}^F(k) \xi(k)]. \end{aligned} \quad (12)$$

in which \mathcal{N} is a normalization constant that again cancels in the cocient of I functions in (3). Note that the propagators now are the usual mass less Feynman ones, but the vertex terms \widehat{V}^{int} include additional contributions associated to the renormalization of the condensate parameters. The vertices expressed in terms of the derivatives over the sources have the form

$$\begin{aligned} &\widehat{V}^{int}[\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{\eta}}, \frac{\delta}{-\delta \eta}, \frac{\delta}{\delta \bar{\xi}}, \frac{\delta}{-\delta \xi}, \frac{\partial}{\partial j(0)}, \frac{\partial}{-\partial \eta(0)}, \frac{\partial}{\partial \bar{\eta}(0)}] \\ &= V^{int}[\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{\eta}}, \frac{\delta}{-\delta \eta}, \frac{\delta}{\delta \bar{\xi}}, \frac{\delta}{-\delta \xi}] + \\ &+ \frac{(Z_3 - 1)}{Z_3} (\frac{2C_g}{(2\pi)^D})^{-1} \frac{\partial^2}{2\partial j_\mu^a(0) \partial j_\mu^a(0)} \\ &+ \frac{(Z_2 - 1)}{Z_2} (\frac{C_q}{(2\pi)^D})^{-1} \frac{\partial^2}{-\partial \eta_u^i(0) \partial \bar{\eta}_u^i(0)}. \end{aligned}$$

Let us search now in what follows for a reordering of the perturbative expansion seeking for explicitly introduce the dimensional transmutation effect in the modified representation [19].

For this purpose a first idea comes from the fact that the auxiliary fields enter as sorts of background constant fields. This fact directly leads to a similar proposal to one made in Ref. [10]. In that work, the modified propagators considered here were first introduced for to be employed in the modifying the perturbative expansion. However, there, it was also discussed an alternative scheme in which the generating functional Z for QCD was considered as an average over constant gluon fields. This superposition allowed to argue that the Fradkin's general functional differential equations for Z [21], should be exactly obeyed by the mean value over constant fields of auxiliary Z functionals associated to arbitrary constant mean fields. As it will be seen from the following discussion the final form of the functional obtained here indicates the similarity between the two proposals advanced in Ref. [10]. However, in that work there was not clarity about the possibility of introducing a weighted average, and thus about how conveniently define it.

The auxiliary fields appear now in the vertices as kinds of constant gluon or quark background fields. Therefore, it seems natural to express the expansion (before the mean value over the auxiliary quantities is taken) in terms of the gluon and quark propagators in the presence of such fields.

For this purpose, from \widehat{V}^{int} in (12) the contribution to its expansion coming from the terms being second order in the gluon and quark fields, but also including auxiliary backgrounds, will be subtracted. These terms, can be now acted on the exponential containing the usual Feynman propagators contracted with the sources. Further, a recourse can be employed of expressing back the exponential of the quadratic forms of the sources in terms of the Feynman propagators in equation (12), as a continual integral over the gluon, quark and ghost fields. Then, the previously mentioned exponential of the quadratic form in the functional derivatives over the gluon and quark fields, simply will produce an additional quadratic form within the exponential of the continual integral. Collecting together the total quadratic form in the fields, leads to a new modified free path integral over which the remaining exponential of the vertices will act. Its expression is

$$\begin{aligned}
Z^{(0)}[j, \eta, \bar{\eta}, \xi, \bar{\xi} | C_q^b, C_g^b] = & \frac{1}{\mathcal{N}} \int \int d\alpha d\bar{\chi} d\chi \exp[-\bar{\chi}_u^i \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}] \int \mathcal{D}[A, \bar{\Psi}, \Psi, \bar{c}, c] \times \\
& \times \exp[- \int \frac{dk}{(2\pi)^D} [\frac{1}{2} A_\mu^a(-k) (\mathbf{D}^{ac} \mathbf{D}^{cb} \delta_{\mu\nu} - \frac{\mathbf{D}_\mu^{ac} \mathbf{D}_\nu^{cb} + \mathbf{D}_\nu^{ac} \mathbf{D}_\mu^{cb}}{2} \\
& + \frac{\delta^{ab}}{\alpha} k_\mu k_\nu) A_\nu^b(k) + \int \frac{dk}{(2\pi)^D} \bar{c}_\mu^a(-k) k_\mu \mathbf{D}_\mu^{ac} c(k) \\
& + \int \frac{dk}{(2\pi)^D} \bar{\Psi}^i(-k) \gamma_\mu \mathbf{D}_\mu^{ij} \Psi^j(k) + \\
& + \int \frac{dk}{(2\pi)^D} \bar{\Psi}^{i,u}(-k) (-g) (\frac{C_q^b}{(2\pi)^D})^{\frac{1}{2}} \gamma_\mu^u{}^v T_a^{ij} \chi^{j,v} A_\nu^a(k) + \\
& + \int \frac{dk}{(2\pi)^D} A_\mu^a(-k) \bar{\chi}^{i,u}(-g) (\frac{C_q^b}{(2\pi)^D})^{\frac{1}{2}} \gamma_\mu^u{}^v T_a^{ij} \Psi^{j,v}(k) + \\
& + \int \frac{dk}{(2\pi)^D} \bar{c}^a k^2 c^b + \int \frac{dk}{(2\pi)^D} (j(-k) A(k) + \bar{\eta}(-k) \Psi(k) + \\
& \bar{\Psi}(k) \eta(-k) + \bar{\xi}(-k) c(k) + \bar{c}(-k) \xi(k))],
\end{aligned}$$

where $\mathbf{D}_\mu^{ij} = k_\mu \delta^{ij} + g(\frac{2C_q}{(2\pi)^D})^{\frac{1}{2}} T_a^{ij} \alpha_\mu^a$ and $\mathbf{D}_\mu^{ab} = k_\mu \delta^{ab} - ig(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}} f^{abc} \alpha_\mu^c$.

The above expression can be converted to a more compact form by defining a composite field and its source and their conjugates, having boson and fermion components, as follows

$$\Phi = \left\{ \begin{array}{c} A_\mu^a \\ \Psi^{r,u} \\ c^a \end{array} \right\}, \Phi^* = \overbrace{\left\{ A_\mu^a \quad \bar{\Psi}^{r,u} \quad \bar{c}^a \right\}} \quad (13)$$

$$J = \left\{ \begin{array}{c} j_\mu^a/2 \\ \eta^{r,u} \\ \xi^a \end{array} \right\} J^* = \overbrace{\left\{ j_\mu^a/2 \quad \bar{\eta}^{r,u} \quad \bar{\xi}^a \right\}} \quad (14)$$

Therefore, a new free generating functional $Z^{(0)}$ can then be expressed in the way

$$Z^{(0)}[j, \eta, \bar{\eta}, \xi, \bar{\xi} | C_q^b, C_g^b] = \frac{1}{\mathcal{N}} \int d\alpha d\bar{\chi} d\chi \exp[-\bar{\chi}_u^i \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}] \times \exp[-\frac{1}{4} V^D F_{\mu\nu}^a(\alpha) F_{\mu\nu}^a(\alpha)] \int \mathcal{D}[\Phi] \exp[\int \frac{dk}{(2\pi)^D} \Phi^*(-k) S^{-1}(k) \Phi(k) + J^*(-k) \Phi(k) + \Phi^*(-k) J(k)] \quad (15)$$

$$= \frac{1}{\mathcal{N}} \int \int d\alpha d\bar{\chi} d\chi \exp[-\bar{\chi}_u^i \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}] \times \exp[-\frac{1}{4} V^D F_{\mu\nu}^a(\alpha) F_{\mu\nu}^a(\alpha)] \quad (16)$$

$$\exp[\int \frac{dk}{(2\pi)^D} \Phi^*(-k) S^{-1}(k) \Phi(k)] \exp[J^*(-k) S(k) J(k)] \quad (17)$$

$$= \frac{1}{\mathcal{N}} \int \int d\alpha d\bar{\chi} d\chi \exp[-\bar{\chi}_u^i \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}] \mathcal{M}(\alpha, \bar{\chi}, \chi) \times \exp[\int \frac{dk}{(2\pi)^D} J^*(-k) S(k) J(k)], \quad (18)$$

The quantity $F_{\mu\nu}^a(\alpha) = g f^{abc} \alpha_\mu^b \alpha_\nu^c$ is the field intensity of the gluon constant field α which Lagrangian appeared due to the shift done in the auxiliary fields. Also, the functional integral differential has been written as

$$\mathcal{D}[\Phi] = \mathcal{D}[A, \bar{\Psi}, \Psi, \bar{c}, c],$$

and the matrix S^{-1} has the block structure

$$S^{-1} = \left\{ \begin{array}{ccc} \mathbf{A}/2 & \mathbf{C} & 0 \\ \mathbf{D} & \mathbf{B} & 0 \\ 0 & 0 & \mathbf{G} \end{array} \right\}, \quad (19)$$

where the matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} and \mathbf{G} are defined by the expressions

$$\begin{aligned} \mathbf{A}^{(\mu,a),(\nu,b)}(\alpha) &\equiv -(\mathbf{D}^{ac} \mathbf{D}^{cb} \delta_{\mu\nu} - \frac{\mathbf{D}_\mu^{ac} \mathbf{D}_\nu^{cb} + \mathbf{D}_\nu^{ac} \mathbf{D}_\mu^{cb}}{2} + \frac{\delta^{ab}}{\alpha} k_\mu k_\nu), \\ \mathbf{B}^{(u,r),(\nu,s)}(\alpha, \bar{\chi}, \chi) &\equiv \gamma_\mu (k_\mu \delta^{ij} + g \beta_\mu^a T_a^{ij}), \\ \mathbf{D}^{(u,r),(\nu,b)}(\bar{\chi}, \chi) &\equiv -g (\frac{C_q^b}{(2\pi)^D})^{\frac{1}{2}} \gamma_v^u T_b^{ij} \chi^{q,t}, \\ \mathbf{C}^{(\mu,a),(\nu,s)}(\bar{\chi}, \chi) &\equiv -g (\frac{C_q^b}{(2\pi)^D})^{\frac{1}{2}} \bar{\chi}^{q,t} \gamma_\mu^q T_b^{ts}, \\ \mathbf{G}^{ab}(\alpha) &= k_\mu \mathbf{D}_\mu^{ab}, \\ \mathbf{D}_\mu^{ab} &= k_\mu \delta^{ab} + g f^{abc} \beta_\mu^c, \\ \beta_\mu^a &= (\frac{2C_g^b}{(2\pi)^D})^{\frac{1}{2}} \alpha_\mu^a. \end{aligned}$$

In expression (16) the function of the auxiliary fields \mathcal{M} is given by

$$\begin{aligned}
\mathcal{M}(\alpha, \bar{\chi}, \chi) &= \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{4} V^D F_{\mu\nu}^a(\alpha) F_{\mu\nu}^a(\alpha)\right] \times \\
&\quad \exp\left[\int \frac{dk}{(2\pi)^D} \Phi^*(-k) S^{-1}(k) \Phi(k)\right] \\
&= \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{4} V^D F_{\mu\nu}^a(\alpha) F_{\mu\nu}^a(\alpha)\right] \times \\
&\quad \int \mathcal{D}[A, \bar{\Psi}, \Psi, \bar{c}, c] \exp\left[\int \frac{dk}{(2\pi)^D} \left(A(-k) \frac{\mathbf{A}(k)}{2} A(k) + \bar{\Psi}(-k) \mathbf{B}(k) \Psi(k) + \right.\right. \\
&\quad \left.\left. A(-k) \mathbf{C}(k) \Psi(k) + \bar{\Psi}(-k) \mathbf{D}(k) A(k) + \bar{c}(-k) \mathbf{G}(k) c(k)\right)\right] \\
&= \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{4} V^D F_{\mu\nu}^a(\alpha) F_{\mu\nu}^a(\alpha)\right] \times \\
&\quad \text{Det}^{-\frac{1}{2}}[\mathbf{A}] \text{Det}[\mathbf{B} - \mathbf{D}\mathbf{A}^{-1}\mathbf{C}] \text{Det}[\mathbf{G}] \\
&= \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{4} V^D F_{\mu\nu}^a(\alpha) F_{\mu\nu}^a(\alpha)\right] \times \\
&\quad \exp\left[-\frac{1}{2} \text{Tr}[\text{Log}[\mathbf{A}]] + \text{Tr}[\text{Log}[\mathbf{B} - \mathbf{D}\mathbf{A}^{-1}\mathbf{C}]] + \text{Tr}[\text{Log}[\mathbf{G}]]\right],
\end{aligned}$$

in which the matrix indices of the fields with the block matrices had not been written explicitly to avoid a more cumbersome expression. However, their restitution seems to be clearly feasible. Henceforth, the following expression can be written for the generating functional Z

$$Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}] = \frac{I[j, \eta, \bar{\eta}, \xi, \bar{\xi}]}{I[0, 0, 0, 0]} \quad (20)$$

$$\begin{aligned}
I[j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \frac{1}{\mathcal{N}} \int \int d\alpha d\bar{\chi} d\chi \exp\left[-\bar{\chi}_u^i \chi_u^i - \frac{\alpha_\mu^a \alpha_\mu^a}{2}\right] \times \mathcal{M}(\alpha, \bar{\chi}, \chi) \\
&\quad \exp\left[\tilde{V}^{int}\left[\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{\eta}}, \frac{\delta}{\delta \eta}, \frac{\delta}{\delta \bar{\xi}}, \frac{\delta}{\delta \xi}\right], \alpha, \bar{\chi}, \chi\right] \times \\
&\quad \exp\left[\int \frac{dk}{(2\pi)^D} J^*(-k) S(k) J(k)\right], \quad (21)
\end{aligned}$$

in which the composite sources J and propagator S where defined in (13) and (19). The vertex terms \tilde{V}^{int} in (20) are equal to the ones in \hat{V}^{int} plus one additional second order in g one which is linear in the gluonic auxiliary field. Its expression is given at the end of Appendix A.

1. Expansion parameters

At this point is useful to recall that the modified perturbative expansion under consideration has a set of three parameters on which the physical quantities depend: (g, C_q, C_g) . All of them will be assumed here to have a dimension defined in powers of the renormalization scale μ . However, the alternative representation (20) suggests a modification of the relevant parameters for the expansion in seeking for the realization of the dimensional transmutation effect [19]. This idea comes from the form of the propagator S in (20). It can be noted that this new free propagator can be made gauge coupling independent simply by defining the new set of independent parameters:

$$\begin{aligned}
g &= g, \\
m_g^2 &= g^2 C_g, \\
m_q^2 &= (g^2 C_q)^{\frac{2}{3}}. \quad (22)
\end{aligned}$$

With this definition, the propagator of the composite field becomes coupling independent and all the vertices non associated to counterterms are mass independent. It can be remarked that these gluon and quark mass parameters where the ones defining the prediction for the constituent masses in [12, 18].

It seem useful to resume here the dimensions of the various fields and constants

$$\begin{aligned} D[A] &= \frac{D-2}{2}, \quad D[\Psi] = \frac{D-1}{2} = D[\bar{\Psi}], \quad D[\chi] = \frac{D-2}{2} = D[\bar{\chi}], \\ D[g] &= 2 - \frac{D}{2}, \quad D[C_g] = D-2, \quad D[C_q] = D-1. \end{aligned}$$

Then, the quark mass parameters m_q having the expression $m_q^2 = (g^2 C_q)^{\frac{2}{3}}$ has a dimension equal to two not changing under the regularization since

$$D[m_q^2] = \frac{2}{3}(4 - D + D - 1) = 2.$$

The same is valid for the gluon mass parameter which dimension is

$$D[m_g^2] = (4 - D + D - s) = 2.$$

2. Connected green Functions generator and Effective Action

The connected Green functions generating functional W and the Effective Action Γ are defined now by the usual Legendre transformation as

$$\begin{aligned} W[j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \log[Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}]], \\ \Gamma[A, \bar{\Psi}, \Psi, \bar{\chi}, \chi] &= W[j, \eta, \bar{\eta}, \xi, \bar{\xi}] - \int dx (j A + \bar{\eta} \Psi + \bar{\Psi} \eta + \bar{\xi} \chi + \bar{\chi} \xi), \end{aligned}$$

in which the mean fields are determined from Z through

$$\begin{aligned} A_\mu^a(x) &= \frac{\delta}{\delta j_\mu^a(x)} \log(Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}]), \\ \Psi(x) &= \frac{\delta}{\delta \bar{\eta}(x)} \log(Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}]), \quad \bar{\Psi}(x) = \frac{\delta}{\delta (-\eta(x))} \log(Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}]), \\ c(x) &= \frac{\delta}{\delta \bar{\xi}(x)} \log(Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}]), \quad \bar{c}(x) = \frac{\delta}{\delta (-\xi(x))} \log(Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}]). \end{aligned}$$

It is important to notice that by the definition of the original Z , W is exactly the sum of all connected diagram in which the lines are the addition of the usual Feynman propagators plus the "condensate propagators". However, after the introduction of the auxiliary fields, the alternative form of Z became a mean value of generating functionals $Z(\alpha, \bar{\chi}, \chi) = \exp(W(\alpha, \bar{\chi}, \chi))$ depending on the auxiliary fields. It can be suspected that the mean value of $Z(\alpha, \bar{\chi}, \chi)$ coincides with the exponentiation of the average of $W(\alpha, \bar{\chi}, \chi)$, and therefore this quantity should be equal with W . However, we have not the proof of this property yet. It will be considered in future extension of the work since some hints point in the direction of its validity, at least approximately in the infinite volume limit.

III. QUARK EFFECTIVE POTENTIAL IN ORDER g^0

Let us consider the one loop Effective Action when only the quark condensate is retained. Then, Γ at zero values of the mean fields and their sources can be written in the form

$$\begin{aligned} \Gamma(m_q) &= \log\left[\frac{Z[0, 0, 0, 0, 0|C_q, 0]}{Z[0, 0, 0, 0, 0|0, 0]}\right], \\ &= \log\left[\int \int d\bar{\chi} d\chi \exp(-\bar{\chi}\chi) \exp\left\{V^{(D)} \int \frac{dk}{(2\pi)^D} Tr_{spin,color}(\log[(k_\mu \gamma_\mu)^{(u,r),(v,s)}] - \right. \right. \\ &\quad \left. \left. \frac{m_q^3}{(2\pi)^D} \gamma_\mu^u T_a^{rt} \chi^{q,t} \frac{1}{(-k^2)} \bar{\chi}^{q',t'} \gamma_\mu^{q'} T_a^{t's} - \log[(k_\mu \gamma_\mu)^{(u,r),(v,s)}])\right\}\right]. \end{aligned}$$

The mean value over the auxiliary field appearing above can be expressed as follows

$$\begin{aligned}\mathcal{G} &= \int \int d\bar{\chi} d\chi \exp(-\bar{\chi}\chi) \exp \left\{ V^D \int \frac{dq}{(2\pi)^D} Tr_{spin,color}(\log[(q_\mu \gamma_\mu)^{(u,r),(v,s)} - \right. \\ &\quad \left. \gamma_\mu^u q T_a^{rt} \chi^{q,t} \frac{1}{(-q^2)} \bar{\chi}^{q',t'} \gamma_\mu^{q'v} T_a^{t's}] - \log[(q_\mu \gamma_\mu)^{(u,r),(v,s)}]) \right\} \\ &= \int \int d\bar{\chi} d\chi \exp(-\bar{\chi}\chi) \exp \left\{ V^D \left(\frac{m_q^3}{(2\pi)^D} \right)^{\frac{D}{3}} \mathcal{F}(D) \right\}.\end{aligned}$$

The factor $\mathcal{F}(D)$ is only dependent on the dimension, and for this pure quark case is convergent due to the high dimension of the parameter m_q^3 . Its expression in terms of an integral over a dimensionless variables takes the form

$$\begin{aligned}\mathcal{F}(D) &= \int \int d\bar{\chi} d\chi \exp(-\bar{\chi}\chi) \exp \left\{ \int \frac{dq}{(2\pi)^D} Tr_{D,C_f}(\log[(q_\mu \gamma_\mu)^{(u,r),(v,s)} - \right. \\ &\quad \left. \gamma_\mu^u q T_a^{rt} \chi^{q,t} \frac{1}{(-q^2)} \bar{\chi}^{q',t'} \gamma_\mu^{q'v} T_a^{t's}] - \log[(q_\mu \gamma_\mu)^{(u,r),(v,s)}]) \right\}.\end{aligned}\quad (23)$$

It seems possible to evaluate \mathcal{G} if the expansion over the auxiliary fermion fields is properly investigated. However, this requires a separate detailed study to be considered elsewhere. Here, we will evaluate it, in a kind of mean field approximation, in which the product of the fields $\chi^{q,t} \bar{\chi}^{q',t'}$ in (23) will be replaced by its "mean" value over the integration of the auxiliary fields. That is

$$\chi^{q,t} \bar{\chi}^{q',t'} \rightarrow \int \int d\alpha d\bar{\chi} d\chi \exp(-\bar{\chi}\chi) \chi^{q,t} \bar{\chi}^{q',t'} = \delta^{qq'} \delta^{tt'}.$$

After the use of the relations :

$$\begin{aligned}T_a^{ik} T_a^{kj} &= C_F \delta^{ij}, \quad C_F = \frac{N^2 - 1}{2N}, \\ \gamma_\mu^u q \gamma_\mu^{qv} &= -D \delta^{uv},\end{aligned}$$

the $\mathcal{F}(D)$ factor gets the simple form

$$\begin{aligned}\mathcal{F}(D) &= \exp \left\{ 4N \int \frac{dq}{(2\pi)^D} \log \left[1 + \frac{4D^2(N^2 - 1)^2}{4N^2} \frac{1}{q^2} \right] \right\} \\ &= \exp \left\{ 4N \frac{\pi^{\frac{D}{2}} D}{\Gamma(\frac{D}{2} + 1)} \int \frac{q^{D-1} dq}{(2\pi)^D} \log \left[1 + \frac{4D^2(N^2 - 1)^2}{4N^2} \frac{1}{(q^2)^3} \right] \right\}.\end{aligned}$$

Therefore, the potential have the expression

$$\begin{aligned}V(m_q) &= -\Gamma(m_q) = -V^D \left(\frac{g^2 C_q}{(2\pi)^D} \right)^{\frac{D}{3}} \mathcal{F}(D) \\ &= -\frac{V^D}{(2\pi)^{\frac{D^2}{3}}} \mu^4 \left(\frac{m_q}{\mu} \right)^D 4N \frac{\pi^{\frac{D}{2}} D}{\Gamma(\frac{D}{2} + 1)} \int \frac{q^{D-1} dq}{(2\pi)^D} \log \left[1 + \frac{4D^2(N^2 - 1)}{4N^2} \frac{1}{(q^2)^3} \right],\end{aligned}$$

which in the limit $D \rightarrow 4$ leads to the energy density $v(m_q)$

$$v(m_q) = \frac{V(m_q)}{V^4} = -\frac{8\pi^2 N m_q^4}{3(2\pi)^{\frac{16}{3}}} \int \frac{q^3 dq}{(2\pi)^4} \log \left[1 + \frac{64(N^2 - 1)^2}{4N^2} \frac{1}{(q^2)^3} \right].$$

The dependence of $v(m_q)$ on the mass parameter m_q is plotted in Fig. 1. As in Ref. [14], the result indicates a dynamical generation of the quark condensate in this simple approximation. The result is unbounded from below. The possibility that higher order corrections could stabilize a minimum will be investigated in future extensions of this work.

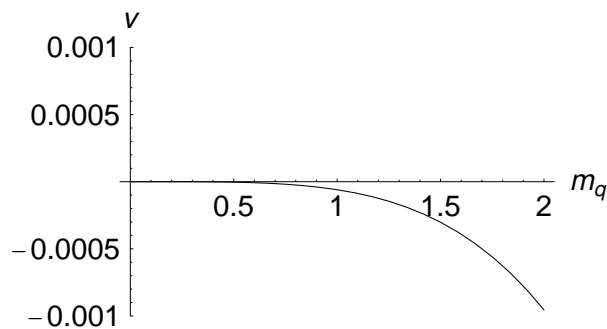


FIG. 1: The energy density estimated in the zeroth order in the coupling approximation, plotted as a function of the quark mass parameter m_q . Note that in this low order approximation, the result indicates a dynamical symmetry breaking under the generation of a quark condensate. The same outcome was obtained in Ref. 13 in which the same result came from a less systematic analysis. The expansion parameters of any quantity are assumed to be the coupling constant g and the quark mass parameter m_q .

IV. SUMMARY

The renormalization of the modified perturbation expansion for mass less QCD proposed in previous works is started to be investigated. A generating functional Z of the finite Green functions is constructed in terms of the renormalized coupling, parameters and fields. Mass counterterms are also introduced assumed their most probable need, since the theory is expected to generate masses for the originally mass less fields. However, the bare masses are assumed to vanish in consistency with the connection of the interaction on a mass less theory which is reflected by the starting unrenormalized generating functional. Expressions for the vertex terms are given. The Z functional is transformed to an alternative representation as a mean value over a class of generating functionals associated to background field theories in presence of constant and homogeneous auxiliary gluon and quark fields. In this variant of the formulation, the gluon and quark field are coupled in a global propagator mixing the boson and fermion fields. The analysis suggests the convenience of introducing as the independent expansion parameters for the physical quantities, the coupling constant g as before, and two mass parameters m_q and m_g . These parameters are simply related with the quark and gluon condensate constants C_q and C_g respectively and retains their dimension equal to two under dimensional regularization. Their relation to the constants reflecting the quark and gluon condensates are $m_q^2 = (g^2 C_q)^{\frac{2}{3}}$ and $m_g^2 = g^2 C_g$. They were relevant in the prediction of the constituent quark masses done in [12, 14].

An evaluation of the lowest order contribution to the Effective Action is presented for the case in which only a quark condensate is retained. The renormalization at this modified tree level was not required. This lower order result indicates a dynamical generation of a quark condensate. This prediction was also obtained in Ref. 14. However, here it is appearing from a more systematic framework. For the extension of the work, it is planned to investigate the possibilities that higher order contributions could produce a minimum of the potential. Such a result might open a way for the application of the modified expansion in justifying a kind of Top condensate model as an effective description of mass less QCD.

Acknowledgments

The invitation and kind hospitality of the High Energy Section of the Abdus Salam International Center for Theoretical Physics (ASICTP) and its Head S. Randjbar-Daemi, allowing for a very helpful visit to the Center, is deeply acknowledged. I express also my gratitude by the support to the work received from the Office of External Activities of ICTP (OEA), through the Network on *Quantum Mechanics, Particles and Fields*(Net-35). The useful remarks during the stay at the AS ICTP received from G. Thompson and K. Narain are also very much appreciated.

APPENDIX A

The explicit form of all the counterterms that appear in (6) is the following

$$\begin{aligned}
V_g^{(1)}[\frac{\delta}{\delta j}] &= \frac{1}{3!} \int \int dk_1 dk_2 dk_3 (2\pi)^D \delta^D(k_1 + k_2 + k_3) \times \\
&\quad V_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(k_1, k_2, k_3) \frac{\delta}{\delta j_{\mu_1}^{a_1}(k_1)} \frac{\delta}{\delta j_{\mu_2}^{a_2}(k_2)} \frac{\delta}{\delta j_{\mu_3}^{a_3}(k_3)}, \\
V_g^{(2)}[\frac{\delta}{\delta j}] &= \frac{1}{4!} \int \int dk_1 dk_2 dk_3 dk_4 (2\pi)^D \delta^D(k_1 + k_2 + k_3 + k_4) \times \\
&\quad V_{\mu_1 \mu_2 \mu_3 \mu_4}^{a_1 a_2 a_3 a_4}(k_1, k_2, k_3, k_4) \frac{\delta}{\delta j_{\mu_1}^{a_1}(k_1)} \frac{\delta}{\delta j_{\mu_2}^{a_2}(k_2)} \frac{\delta}{\delta j_{\mu_3}^{a_3}(k_3)} \frac{\delta}{\delta j_{\mu_4}^{a_4}(k_4)}, \\
V_q^{(1)}[\frac{\delta}{\delta \bar{\eta}}, \frac{\delta}{-\delta \eta}, \frac{\delta}{\delta j}] &= \int \int dk_1 dk_2 dk_3 (2\pi)^D \delta^D(k_1 + k_2 + k_3) \\
&\quad \frac{\delta}{\delta \eta^{i_1}(k_1)} g \gamma_{\mu_3} T_{a_3}^{i_1 i_2} \frac{\delta}{\delta \bar{\eta}^{i_2}(k_2)} \frac{\delta}{\delta j_{\mu_3}^{a_3}(k_3)}, \\
V_{gh}^{(1)}[\frac{\delta}{\delta \bar{\xi}}, \frac{\delta}{-\delta \xi}, \frac{\delta}{\delta j}] &= \int \int dk_1 dk_2 dk_3 (2\pi)^D \delta^D(k_1 + k_2 + k_3) \\
&\quad \frac{\delta}{\delta \eta^{a_2}(k_2)} (-ig) k_{2\mu_3} f^{a_1 a_2 a_3} \frac{\delta}{\delta \bar{\xi}^{a_3}(k_3)} \frac{\delta}{\delta j_{\mu_3}^{a_3}(k_3)}, \\
V_g^{(0)}[\frac{\delta}{\delta j}] &= \frac{1}{3!} \int \int dk_1 dk_2 (2\pi)^D \delta^D(k_1 + k_2) \times \\
&\quad \frac{\delta}{\delta j_{\mu_2}^{a_2}(k_2)} \frac{(Z_3 - 1)}{2} \delta^{a_2 a_1} (k_1^2 \delta_{\mu_2 \mu_1} - k_{1\mu_2} k_{1\mu_1}) \frac{\delta}{\delta j_{\mu_1}^{a_1}(k_1)}, \\
V_q^{(0)}[\frac{\delta}{\delta \bar{\eta}}, \frac{\delta}{-\delta \eta}] &= \int \int dk_1 dk_2 (2\pi)^D \delta^D(k_1 + k_2) \\
&\quad \frac{\delta}{\delta \eta(k_2)} [(Z_2 - 1) k_{1\mu} \gamma_\mu + \delta m (g_0, C_q^0, C_g^0)] \frac{\delta}{\delta \bar{\eta}(k_1)}, \\
V_{gh}^{(0)}[\frac{\delta}{\delta \bar{\xi}}, \frac{\delta}{-\delta \xi}] &= \int \int dk_1 dk_2 (2\pi)^D \delta^D(k_1 + k_2) \\
&\quad \frac{\delta}{\delta \xi^{a_2}(k_2)} (Z_3 - 1) \delta^{a_2 a_1} k_1^2 \frac{\delta}{\delta \bar{\xi}^{a_1}(k_1)}.
\end{aligned}$$

The only new vertex that appears in \tilde{V}^{int} in (20) after the quadratic form in the gluon and quark fields depending on the auxiliary fields is extracted in order to arrive to this expression (20), has the form: .

$$\begin{aligned}
V_g^{(2)}[\frac{\delta}{\delta j}] &= \frac{1}{4!} \int \int dk_1 dk_2 dk_3 dk_4 (2\pi)^D \delta^D(k_1 + k_2 + k_3 + k_4) \times \\
&\quad V_{\mu_1 \mu_2 \mu_3 \mu_4}^{a_1 a_2 a_3 a_4}(k_1, k_2, k_3, k_4) \left(\frac{\delta}{\delta j_{\mu_1}^{a_1}(k_1)} + 4 \left(\frac{2C_g}{(2\pi)^D} \right)^{\frac{1}{2}} \alpha_{\mu_1}^a \delta^D(k_1) \right) \frac{\delta}{\delta j_{\mu_2}^{a_2}(k_2)} \frac{\delta}{\delta j_{\mu_3}^{a_3}(k_3)} \frac{\delta}{\delta j_{\mu_4}^{a_4}(k_4)}.
\end{aligned}$$

-
- [1] E. Shuryak, *The QCD vacuum, hadrons and superdense matter*, (World Scientific, Singapore, 1988).
[2] T. Schafer and E. V. Shuryak, Rev. of Mod. Phys. **70**, 323 (1998).
[3] H. Fritzsch and D. Holtmannspotter, Phys. Lett. **B338**, 290 (1994).
[4] G.K. Savvidy, Phys. Lett. **B71**, 133 (1977), I. Batalin, S.G. Matinyan and G.K. Savvidy, Sov. J. Nucl. **41**, 214 (1977).

- [5] J.M. Cornwall, R. Jackiw and E. Tomboulis, Phys. Rev. D**10**, 2428 (1974).
- [6] V.A. Miransky, in *Nagoya Spring School on Dynamical Symmetry Breaking*, ed. K. Yamawaki, World Scientific (1991).
- [7] J.M. Cornwall, Phys. Rev. D**26**, 1453 (1982).
- [8] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. D**41**, 1647 (1990).
- [9] A. Cabo, O.K. Kalashnikov and A.E. Shabad, Nucl.Phys. B**185**, 473 (1981).
- [10] A. Cabo, S. Peñaranda and R. Martínez, Mod. Phys. Lett. A**10**, 2413 (1995).
- [11] M. Rigol and A. Cabo, Phys. Rev. D**62**, 074018 (2000).
- [12] A. Cabo and M. Rigol, Eur. Phys. J. C**23**, 289 (2002).
- [13] A. Cabo and M. Rigol, Eur. Phys. J. C**47**, 95 (2006).
- [14] A. Cabo and D. Martinez-Pedrera, Eur. Phys. J. C**47**, (2006).
- [15] P. Hoyer, NORDITA-2002-19 HE (2002), e-Print Archive: hep-ph/0203236 (2002).
- [16] P. Hoyer, *Proceedings of the ICHEP* 367-369, Amsterdam (2002), e-Print Archive: hep-ph/0209318.
- [17] P. Hoyer, Acta Phys. Polon. B**34**, 3121 (2003). e-Print Archive: hep-ph/0304022.
- [18] A. Cabo, JHEP **04**, 044 (2003).
- [19] S. Coleman and E. Weinberg, Phys. Rev. D**7**, 1888 (1973).
- [20] G. Leibbrandt, Rev. Mod. Phys **47**, 849 (1975).
- [21] E. S. Fradkin, *Quantum Field Theory and Hydrodynamics*, Series Lebedev Physics Institute (Trudy Lebedev) V. **29**, Consultants Bureau, New York, 1967.